8.4 Laws of Logarithms

A Power Law

$$\log_b x^n = n\log_b x, \quad x > 0$$

Ex 1. Prove the power law.

Ex 2. Use the power law to simplify.

a) $\log_2 64$

b) $\log 0.0001$

c) $\ln\sqrt[3]{e^2}$

Ex 3. Simplify. State restrictions.

a) $\log x^2$

b) $\ln\sqrt[5]{x^4}$

c) $\log_2 x^{1/3}$

B Product Law

$$\log_b (xy) = \log_b x + \log_b y; \quad x, y > 0$$

Ex 4. Prove the product law.

Ex 5. Use the power law and the product law to expand. State restrictions.

a) $\log(10xy)$

b) $\log_2(16a^2b^3)$

Ex 6. Write as a single logarithm. Evaluate, if possible. State restrictions.

a) $\log 20 + \log 50 + \log 0.1$

b) $2\log_5 10 + \frac{1}{2}\log_5 \frac{1}{2}$

c) $2\ln x + 3\ln y + \frac{1}{3}\ln z$

C Quotient Law

$$\log_b \frac{x}{y} = \log_b x - \log_b y; \quad x, y > 0$$

Ex 7. Prove the quotient law.

Ex 8. Expand using the logarithms laws. State restrictions. a) $\log\frac{2}{3}$ b) $\ln\frac{a^2\sqrt{b}}{c^3}$	Ex 9. Write as a single logarithm. Evaluate, if possible. State restrictions. a) $\log_3 18-\log_3 2$ b) $\frac{1}{3}\ln a-\frac{2}{3}\ln b+2\ln 3$
D Change of Base Law $\log_a x=\frac{\log_b x}{\log_b a}$	Ex 10. Prove the change of base law.
Ex 11. Prove the following formulas. a) $\ln 10=\frac{1}{\log e}$ b) $\log_b x=\frac{\log x}{\log b}=\frac{\ln x}{\ln b}$	Ex 12. Use technology to evaluate. a) $\log_2 3$ b) $\log_{\sqrt{3}}\sqrt[5]{2}$
E Change of Base Formula for Exponential Function $a^x=b^{x\log_b a}$	Ex 13. Prove the change of base for exponential function law.
Ex 14. Prove the following relations. a) $x=e^{\ln x}$ b) $x=\ln e^x$ c) $a^x=e^{x\ln a}$ d) $x=b^{\log_b x}$	Ex 15. Change to the base 10. a) $f(x)=2^x$ b) $f(x)=3e^{2x}-1$

Reading: Nelson Textbook, Pages 469-474
Homework: Nelson Textbook, Page 475: #4ab, 5, 6abc, 7d, 8, 9af, 10af, 11af, 12, 15, 17